Teaching and Measuring the Common Core’s Mathematical Practices

Sponsored by: Discovery Education | Math Techbook
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Assistant editor, *Education Week*

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Teaching and Measuring the Common Core’s Mathematical Practices

Expert Presenters:

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Melissa Waggoner, mathematics instructional support teacher and instructional team leader, Thomas Viaduct Middle School, Hanover, Md.

Related Articles:
Engaging Math Students in Productive Struggle
Finding Overlap in the Common Math, Language Arts, and Science Standards
An on-demand archive of this webinar will be available at www.edweek.org/go/webinar in less than 24 hrs.
Session Outcomes

Discuss ways in which teachers can engage ALL students in the behaviors that support the MPs and lead to mathematical proficiency.

Examine methods and tools that are being used to monitor/measure student engagement and growth with the MPs.

Discuss the pros and cons of using these tools in the mathematics classroom.
Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Source: CCSSO & NGA, 2010

FLASHBACK: The Standards for Mathematical Practice...
Well accepted as the starting point into CCSS Implementation

FLASHBACK: The Standards for Mathematical Practice... well accepted as the “starting point” for CCSS Implementation.

We could be implementing the ‘Mathematical Practices’ with (and even regardless of) our current [course, grade level, district or state] standards...

“SMPs” & “CCSS-M” (Interest Over Time)
Google Search: “Standards for Mathematical Practice” – 16,300,000+ results!
Standards for Mathematical Practice...
A Different Look (Gr. 6-12 Howard County, MD)

Overarching habits of mind of a productive mathematical thinker

1. Make sense of problems and persevere in solving them. 47%
6. Attend to precision. 39%

Reasoning & Explaining
2. Reason abstractly and quantitatively. 55%
3. Construct viable arguments and critique the reasoning of others 38%

Modeling & Using Tools
4. Model with mathematics. 40%
5. Use appropriate tools strategically. 49%

Seeing Structure & Generalizing
7. Look for and make use of structure. 21%
8. Look for and express regularity in repeated reasoning. 17%

As of June 16, 2011: N = 402; 90% - classrooms with students exhibiting 1 or more Mathematical Practices, 10% - no evidence of students exhibiting any Practice

Adapted from Tools for the Common Core (2010)
Reflect & Share: What have you done, specifically during the most recent 5 lessons (or days), to support the development of the Mathematical Practices with students?
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years.

The Standards for Mathematical Content support a balanced combination of procedural fluency and conceptual understanding.

In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

Source: CCSSO & NGA, 2010, pp. 6-8
NCTM’s (Process) Standards

- Problem Solving
- Reasoning and Proof
- Communication
- Connections
- Representation

Source: NCTM, 2000
Strategic competence: ability to formulate, represent, and solve mathematical problems.

Conceptual understanding: comprehension of mathematical concepts, operations, and relations.

Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.

Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification.

Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.

Source: Adding It Up, 2001
### (How the 3 Fit Together)

<table>
<thead>
<tr>
<th>NCTM Processes</th>
<th><strong>CCSS – Standards for Mathematical Practice</strong></th>
<th><strong>Adding it Up – Strands of Mathematical Proficiency</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem Solving</td>
<td><em>Make sense of problems and persevere in solving them.</em></td>
<td>Strategic competence</td>
</tr>
<tr>
<td>Reasoning and Proof</td>
<td><em>Reason abstractly and quantitatively.</em></td>
<td>Adaptive reasoning</td>
</tr>
<tr>
<td>Reasoning and Proof</td>
<td><em>Construct viable arguments and critique the reasoning of others.</em></td>
<td>Adaptive reasoning</td>
</tr>
<tr>
<td>Connections</td>
<td><em>Model with mathematics.</em></td>
<td>Strategic competence</td>
</tr>
<tr>
<td>Representation</td>
<td><em>Use appropriate tools strategically.</em></td>
<td>Strategic competence, Conceptual understanding</td>
</tr>
<tr>
<td>Communication</td>
<td><em>Attend to precision.</em></td>
<td>Procedural fluency.</td>
</tr>
<tr>
<td>Connections</td>
<td><em>Look for and make use of structure.</em></td>
<td>Strategic competence</td>
</tr>
<tr>
<td>Reasoning and Proof</td>
<td><em>Look for and express regularity in repeated reasoning.</em></td>
<td>Adaptive reasoning</td>
</tr>
</tbody>
</table>

*Productive disposition*
“Students Who Are College and Career Ready…”

<table>
<thead>
<tr>
<th>Demonstrate independence</th>
<th>Make sense of problems and persevere in solving them</th>
</tr>
</thead>
<tbody>
<tr>
<td>Build strong content knowledge</td>
<td>Reason abstractly and quantitatively</td>
</tr>
<tr>
<td>Respond to the varying demands of audience, task, purpose, and discipline</td>
<td>Construct viable arguments and critique the reasoning of others</td>
</tr>
<tr>
<td>Comprehend as well as critique</td>
<td>Model with mathematics</td>
</tr>
<tr>
<td>Value evidence</td>
<td>Use appropriate tools strategically</td>
</tr>
<tr>
<td>Use technology and digital media strategically and capably</td>
<td>Attend to precision</td>
</tr>
<tr>
<td>Come to understand other perspectives and cultures</td>
<td>Look for and make use of structure</td>
</tr>
<tr>
<td></td>
<td>Look for and express regularity in repeated reasoning</td>
</tr>
</tbody>
</table>

Taken from *Common Core State Standards for English Language Arts and Mathematics*

Source: Howard County Public Schools, August 2010
“Mathematically proficient students...”

SMP 1: Explain, make conjectures, persevere...

SMP 2: Decontextualize* and contextualize ...

SMP 3: Understand, present and critique...

SMP 4: Represent, apply and interpret...

SMP 5: Consider, select...

SMP 6: Communicate precisely...

SMP 7: Discern, recognize and use...

SMP 8: Notice, pay attention to and generalize...
# Standards for Mathematical Practices - “Student Look-fors”

<table>
<thead>
<tr>
<th>Mathematical Topic(s):</th>
<th>Course/Period:</th>
<th>Start/End Times:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. Make sense of problems and persevere in solving them</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Understand the meaning of the problem and look for entry points to its solution</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Analyze information (givens, constraints, relationships, goals)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Make conjectures and plan a solution pathway</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Monitor and evaluate the progress and change course as necessary</td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Check answers to problems and ask, “Does this make sense?”</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Comments:**

| **2. Reason abstractly and quantitatively** |  |  |
| - Make sense of quantities and relationships in problem situations |  |  |
| - Represent abstract situations symbolically and understand the meaning of quantities |  |  |
| - Create a coherent representation of the problem at hand |  |  |
| - Consider the units involved |  |  |
| - Flexibly use properties of operations |  |  |

**Comments:**

| **3. Construct viable arguments and critique the reasoning of others** |  |  |
| - Use definitions and previously established causes/effects (results) in constructing arguments |  |  |
| - Make conjectures and use counterexamples to build a logical progression of statements to explore and support their ideas |  |  |
| - Communicate and defend mathematical reasoning using objects, drawings, diagrams, actions |  |  |
| - Listen to or read the arguments of others |  |  |
| - Decide if the arguments of others make sense and ask probing questions to clarify or improve the arguments |  |  |

**Comments:**

| **4. Model with mathematics.** |  |  |
| - Apply prior knowledge to solve real world problems |  |  |
| - Identify important quantities and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas |  |  |
| - Make assumptions and approximations to make problem simpler |  |  |
| - Check to see if an answer makes sense within the context of a situation and change a model when necessary |  |  |

**Comments:**

| **5. Use appropriate tools strategically.** |  |  |
| - Make sound decisions about the use of specific tools. Examples might include: |  |  |
| - Calculator |  |  |
| - Concrete models |  |  |
| - Digital Technology |  |  |
| - Pencil/paper |  |  |
| - Ruler, compass, protractor |  |  |
| - Use technological tools to visualize the results of assumptions, explore consequences and compare predications with data |  |  |
| - Identify relevant external math resources (digital content on a website) and use them to pose or solve problems |  |  |
| - Use technological tools to explore and deepen understanding of concepts |  |  |

**Comments:**

| **6. Attend to precision.** |  |  |
| - Communicate precisely using clear definitions |  |  |
| - State the meaning of symbols, carefully specifying units of measure, and providing accurate labels |  |  |
| - Calculate accurately and efficiently, expressing numerical answers with a degree of precision |  |  |
| - Provide carefully formulated explanations |  |  |
| - Label accurately when measuring and graphing |  |  |

**Comments:**

| **7. Look for and make use of structure.** |  |  |
| - Look for patterns or structure, recognizing that quantities can be represented in different ways |  |  |
| - Recognize the significance in concepts and models and use the patterns or structure for solving related problems |  |  |
| - View complicated quantities both as single objects or compositions of several objects and use operations to make sense of problems |  |  |

**Comments:**

| **8. Look for and express regularity in repeated reasoning** |  |  |
| - Notice repeated calculations and look for general methods and shortcuts |  |  |
| - Continually evaluate the reasonableness of intermediate results (comparing estimates) while attending to details and make generalizations based on findings |  |  |

**Comments:**

**Additional notes:**

<table>
<thead>
<tr>
<th>Non-evaluative visitor(s):</th>
<th>Date:</th>
</tr>
</thead>
</table>

Source: Elementary Mathematics Specialists & Teacher Leaders Project (2010)
## Standards for Mathematical Practice - Student “Look-fors”

### Overarching habits of mind of a productive mathematical thinker

1. Make sense of problems and persevere in solving them
   - Understand the meaning of the problem and look for entry points to its solution
   - Analyze information (givens, constraints, relationships, goals)
   - Make conjectures and plan a solution pathway
   - Monitor and evaluate the progress and change course as necessary
   - Check answers to problems and ask, “Does this make sense?”

2. Reason abstractly and quantitatively
   - Make sense of quantities and relationships in problem situations
   - Represent abstract situations symbolically and understand the meaning of quantities
   - Create a coherent representation of the problem at hand
   - Consider the units involved
   - Flexibly use properties of operations

3. Construct viable arguments and critique the reasoning of others
   - Use definitions and previously established causes/effects (results) in constructing arguments
   - Make conjectures and use counterexamples to build a logical progression of statements to explore and support their ideas
   - Communicate and defend mathematical reasoning using objects, drawings, diagrams, actions
   - Listen to or read the arguments of others
   - Decide if the arguments of others make sense and ask probing questions to clarify or improve the arguments

4. Model with mathematics
   - Apply prior knowledge to solve real world problems
   - Identify important quantities and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas
   - Make assumptions and approximations to make a problem simpler
   - Check to see if an answer makes sense within the context of a situation and change a model when necessary

5. Use appropriate tools strategically
   - Make sound decisions about the use of specific tools.
     (Examples might include: calculator, concrete models, digital technology, pencil/paper, ruler, compass, protractor.)
   - Use technological tools to visualize the results of assumptions, explore consequences and compare predications with data
   - Identify relevant external math resources (digital content on a website) and use them to pose or solve problems
   - Use technological tools to explore and deepen understanding of concepts

6. Attend to precision.
   - Communicate precisely using clear definitions
   - State the meaning of symbols, carefully specifying units of measure, and providing accurate labels
   - Calculate accurately and efficiently, expressing numerical answers with a degree of precision
   - Provide carefully formulated explanations
   - Label accurately when measuring and graphing

### Comments:

### Reasoning and explaining

### Modeling and using tools

### Seeing structure and generalizing

7. Look for and make use of structure
   - Look for patterns or structure, recognizing that quantities can be represented in different ways
   - Recognize the significance in concepts and models and use the patterns or structure for solving related problems
   - View complicated quantities both as single objects or compositions of several objects and use operations to make sense of problems

8. Look for and express regularity in repeated reasoning
   - Notice repeated calculations and look for general methods and shortcuts
   - Continually evaluate the reasonableness of intermediate results (comparing estimates) while attending to details and make generalizations based on findings

### Comments:
## Standards for Mathematical Practice

<table>
<thead>
<tr>
<th>Key student dispositions:</th>
<th>Teacher actions that will engage students in practices:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Comments:**

*All indicators are not necessary for providing full evidence of practice(s). Each practice may not be evident during every lesson.*

NCSM Summer Leadership Academy • Draft 2011
<table>
<thead>
<tr>
<th>Mathematics Practices</th>
<th>Students:</th>
<th>Teachers:</th>
<th>Comments:</th>
</tr>
</thead>
</table>
| 1. Make sense of problems and persevere in solving them | - Understand the meaning of the problem and look for entry points to its solution  
- Analyze information (givens, constrains, relationships, goals)  
- Make conjectures and plan a solution pathway  
- Monitor and evaluate the progress and change course as necessary  
- Check answers to problems and ask. “Does this make sense?” | - Involve students in rich problem-based tasks that encourage them to persevere in order to reach a solution  
- Provide opportunities for students to solve problems that have multiple solutions  
- Encourage students to represent their thinking while problem solving | Comments: |
| 2. Reason abstractly and quantitatively | - Make sense of quantities and relationships in problem situations  
- Represent abstract situations symbolically and understand the meaning of quantities  
- Create a coherent representation of the problem at hand  
- Consider the units involved  
- Flexibly use properties of operations | - Facilitate opportunities for students to discuss or use representations to make sense of quantities and their relationships  
- Encourage the flexible use of properties of operations, objects, and solution strategies when solving problems  
- Provide opportunities for students to decontextualize (abstract a situation) and/or contextualize (identify referents for symbols involved) the mathematics they are learning | Comments: |
| 3. Construct viable arguments and critique the reasoning of others | - Use definitions and previously established causes/effects (results) in constructing arguments  
- Make conjectures and use counterexamples to build a logical progression of statements to explore and support ideas  
- Communicate and defend mathematical reasoning using objects, drawings, diagrams, and/or actions  
- Listen to or read the arguments of others  
- Decide if the arguments of others make sense and ask probing questions to clarify or improve the arguments | - Provide and orchestrate opportunities for students to listen to the solution strategies of others, discuss alternative solutions, and defend their ideas  
- Ask higher-order questions which encourage students to defend their ideas  
- Provide prompts that encourage students to think critically about the mathematics they are learning | Comments: |
<table>
<thead>
<tr>
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<th>Students:</th>
<th>Teacher(s):</th>
</tr>
</thead>
</table>
| 4. Model with mathematics | - Apply prior knowledge to solve real world problems  
- Identify important quantities and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and/or formulas  
- Use assumptions and approximations to make a problem simpler  
- Check to see if an answer makes sense within the context of a situation and change a model when necessary  
| Comments: | - Use mathematical models appropriate for the focus of the lesson  
- Encourage student use of developmentally and content-appropriate mathematical models (e.g., variables, equations, coordinate grids)  
- Remind students that a mathematical model used to represent a problem’s solution is ‘a work in progress,’ and may be revised as needed  
| Comments: |
| 5. Use appropriate tools strategically | - Make sound decisions about the use of specific tools (Examples might include: calculator, concrete models, digital technologies, pencil/paper, ruler, compass, protractor)  
- Use technological tools to visualize the results of assumptions, explore consequences, and compare predictions with data  
- Identify relevant external math resources (digital content on a website) and use them to pose or solve problems  
- Use technological tools to explore and deepen understanding of concepts  
| Comments: | - Use appropriate physical and/or digital tools to represent, explore and deepen student understanding  
- Help students make sound decisions concerning the use of specific tools appropriate for the grade level and content focus of the lesson  
- Provide access to materials, models, tools and/or technology-based resources that assist students in making conjectures necessary for solving problems  
| Comments: |
| 7. Look for and make use of structure | - Look for patterns or structure, recognizing that quantities can be represented in different ways  
- Recognize the significance in concepts and models and use the patterns or structure for solving related problems  
- View complicated quantities both as single objects or compositions of several objects and use operations to make sense of problems  
| Comments: | - Engage students in discussions emphasizing relationships between particular topics within a content domain or across content domains  
- Recognize that they quantitative relationships modeled by operations and their properties remain important regardless of the operational focus of a lesson  
- Provide activities in which students demonstrate their flexibility in representing mathematics in a number of ways e.g., $76 = (7 \times 10) + 6$; discussing types of quadrilaterals, etc.  
| Comments: |
| 8. Look for and express regularity in repeated reasoning | - Notice repeated calculations and look for general methods and shortcuts  
- Continually evaluate the reasonableness of intermediate results (comparing estimates), while attending to details, and make generalizations based on findings  
| Comments: | - Engage students in discussion related to repeated reasoning that may occur in a problem’s solution  
- Draw attention to the prerequisite steps necessary to consider when solving a problem  
- Urge students to continually evaluate the reasonableness of their results  
| Comments: |
iPhone / iPad App:
Common Core Look4s (CCL4s)
Problems Worth Solving

Launch Event — January 8, 1PM EST
DiscoveryEducation.com/ProblemsWorthSolving

- Engages students with real-world examples and interactive tools
- Uses an inquiry-based instructional approach
- Built on the rigor of the Common Core State Standards
- Embodies the Standards for Mathematical Practice
How can we get our “developing student practitioners of the discipline of mathematics” to take on OWNERSHIP of the MPs as they engage with mathematical content?
A view in & around mathematics classrooms

Consider the...

of each tool or resource.
Teaching the MP’s

1. Make sense of problems and persevere in solving them
   - Make meaning of a problem and look for entry points to its solution
   - Make conjectures about the meaning of a solution
   - Develop a plan
   - Monitor and evaluate progress and change course if necessary
   - Check answers to see if they make sense

<table>
<thead>
<tr>
<th>How Can I Work Like a Mathematician?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard of Math Practice</strong></td>
</tr>
<tr>
<td>--------------------------------------</td>
</tr>
<tr>
<td>1.) Make sense of problems and persevere in solving them.</td>
</tr>
<tr>
<td>2.) Reason abstractly and quantitatively.</td>
</tr>
<tr>
<td>3.) Construct viable arguments and critique the reasoning of others.</td>
</tr>
<tr>
<td>4.) Model with mathematics.</td>
</tr>
</tbody>
</table>

Source: J. Haney (2013)
## How Can I Work Like a Mathematician?

<table>
<thead>
<tr>
<th>Standard of Math Practice</th>
<th>What It Means to Me</th>
<th>How can I demonstrate this as I work</th>
<th>Drawing to Help me Remember</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.) Make sense of problems and persevere in solving them.</td>
<td>Make a plan and carry it out to find an answer.</td>
<td>Organize persevere check</td>
<td>$3 + 9$ organize</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$3 + 9 = 12$ persevere</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$12 - 3 = a$ check</td>
</tr>
<tr>
<td>2.) Reason abstractly and quantitatively.</td>
<td>Connecting math to everyday life.</td>
<td>Using context clues.</td>
<td>$\sqrt{4 + 5 \cdot 6} - 3$ pemdas</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$2 + 30 - 3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$32 - 3$</td>
</tr>
<tr>
<td>3.) Construct viable arguments and critique the reasoning of others.</td>
<td>Critique others work to come up with a final answer.</td>
<td>Compare work.</td>
<td>$2 \neq 3$</td>
</tr>
<tr>
<td>Standard of Math Practice</td>
<td>What It Means to Me</td>
<td>How can I demonstrate this as I work</td>
<td>Drawing to Help me Remember</td>
</tr>
<tr>
<td>---------------------------</td>
<td>--------------------</td>
<td>--------------------------------------</td>
<td>-----------------------------</td>
</tr>
<tr>
<td>5.) Use appropriate tools strategically.</td>
<td>use math tools that will help solve the question</td>
<td>Think about math tools such as a grid, ruler, or scale</td>
<td>![Image]</td>
</tr>
<tr>
<td>6.) Attend to precision.</td>
<td>communicate ideas to accurately answer the question</td>
<td>Get an answer and check with others</td>
<td>( x = ) ( a) 11 ) ( b) 12 ) ( c) 13 ) ( d) 14 )</td>
</tr>
<tr>
<td>7.) Look for and make use of structure.</td>
<td>understand measurements and dimensions</td>
<td>By using scale and measurements</td>
<td>![Image]</td>
</tr>
<tr>
<td>8.) Look for and express regularity in repeated reasoning.</td>
<td>Look for short cuts</td>
<td>Look for any repeated calculations</td>
<td>( \frac{111}{51555} ) ( = ) ( \frac{11}{5155} ) ( \frac{1}{5} ) ( \frac{1}{5} )</td>
</tr>
<tr>
<td>Did I Persevere in Here? Please rate your efforts honestly.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------------------------------------------------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1=“Not really”, 5=“Yes, I definitely did!”</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>o I thought about the problem so that I understood the question(s)</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>o I asked questions to clarify</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>o I used what I knew to help me with what I didn’t know</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>o I used an easier problem to help with something that was harder.</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>o I used more than one strategy.</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>o I drew a picture or created a representation to show my thinking</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>o I asked myself, “Does my answer make sense?”</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>o I checked my work.</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>o I kept trying, even if the problem was challenging!</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Source: C. Therault (2012)
Math Practices Follow-Up

Choose 2 of the practices that you used today.
Explain how you used each practice in 1-2 sentences.

Appropriate tools
Construct viable arguments
Persevere in solving
Repeated reasoning
Reason abstractly/quantitatively

Attend to precision
Model with math
Make use of structure

1)

2)
## Student Reflection – Tool 4

<table>
<thead>
<tr>
<th>CONTENT STANDARD</th>
<th>DATES</th>
<th>ACTIVITIES TO SUPPORT MEETING THE STANDARD</th>
<th>PRACTICES THAT I USED TO MEET THE STANDARD</th>
<th>MY LEVEL OF UNDERSTANDING 1(GOOD)-4(NEED LOTS OF SUPPORT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.EE.A. Use properties of operations to generate equivalent expressions.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Apply properties of operations as strategies to add, subtract, factor, and to expand linear expressions with rational coefficients.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.EE.B. Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: J. Haney (2014)
Infused MP Example 1

An ant has a mass of approximately $4 \times 10^{-3}$ grams and an elephant has a mass of approximately 8 metric tons ($8 \times 10^6$ grams).

1. Determine how many ants it takes to have the same mass as an elephant.

2. Which math practice did you use throughout this task? Explain with specific examples.

Source: M. Waggoner adapted from Illustrative Mathematics (2014)
After joining with another group, compare the two functions that were developed through your research. Compare three aspects of the functions and their representations.

- The other groups' increments went up by ones, but ours went up by threes, and also their x-axis represented number of years, but ours represented number of concerts.
- The other group's equation was different from my group's because we multiplied and subtracted, but they subtracted two numbers in parentheses and multiplied it by x.
- The other groups' x-labels went up to 11, but ours went up to at least 21, and their y-labels went up by millions, and ours went up by millions.

Choose two of the mathematical practices and explain how you used them in this project.

<table>
<thead>
<tr>
<th>Math Practice:</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Make sense of problems and persevere in solving them. We used this mathematical practice in our project because we kept track of our work, we checked if our equation &amp; function tables were correct and we evaluated our graph to see if we were successful.</td>
<td>Attend to precision. We used mathematical practice #6 in our project because we checked over our work to see if our equation was correct and to see if our work was correct. We used precision when solving our problems.</td>
</tr>
</tbody>
</table>

Source: A. Weyforth (2014)
“SMP Role Cards”

Role: SMP #1 Leader
Make sense of problems and persevere in solving them

As SMP #1 Leader, I will:
• Explain what the problem means in my own words
• Encourage my teammates to persevere
• Continue to ask myself and others, “does this make sense?”

Role: SMP #6 Leader
Attend to precision

As SMP #6 Leader, I will:
• Ensure calculations are accurate
• Encourage the use of appropriate vocabulary
• Specify units of measure and label any diagrams to clarify meaning

Source: M. Waggoner (2013)
“SMP Role Cards”

Role: SMP #2 Leader
Reason abstractly and quantitatively

As SMP #2 Leader, I will:
- Create a clear representation of the problem
- Consider the units of measure involved
- Help my teammates to contextualize or decontextualize the problem

Role: SMP #3 Leader
Construct viable arguments and critique the reasoning of others

As SMP #3 Leader, I will:
- Listen to the conclusions of others and decide if they make sense
- Encourage my teammates to respectfully identify flawed logic
- Ask questions to clarify other ideas and conclusions

Source: M. Waggoner (2013)
“SMP Role Cards”

Role: SMP #4 Leader
Model with mathematics

As SMP #4 Leader, I will:
• Identify important relationships in a problem
• Represent the relationship using tools
• Lead the discussion on whether the results make sense, improving the model if necessary

Role: SMP #5 Leader
Use appropriate tools strategically

As SMP #5 Leader, I will:
• Advocate for the appropriate tools from the teacher
• Encourage the correct use of tools
• Recognize when a tool might be beneficial in solving a problem

Source: M. Waggoner (2013)
“SMP Role Cards”

Role: SMP #7 Leader
Look for and make use of structure

As SMP #7 Leader, I will:
- Look closely to see a pattern or a structure in a math situation
- Break down a complicated problem into single objects
- Encourage my teammates to shift perspective if needed

Role: SMP #8 Leader
Look for and express regularity in repeated reasoning

As SMP #8 Leader, I will:
- Notice if calculations are repeated
- Look for a more efficient method to solve problems
- Lead the discussion on making a generalization based on results

Source: M. Waggoner (2013)
Promoting & Messaging...

I make sense of problems
I persevere in solving problems

I am a Math Student
(and I will be college and career ready)

Source: M. Waggoner (2013)
HCPSS District Teacher Evaluation Model

Charlotte Danielson’s Framework For Teaching and Learning
Professional Practice (50%)

Domain 1
Planning and Preparation 12.5%

Domain 2
Classroom Environment 12.5%

Domain 3
Instruction 12.5%

Domain 4
Professional Responsibilities 12.5%

Domain 5
Student Growth (50%)

Qualitative Measures

Quantitative Measures:
Student Learning Objectives and MSA

Literacy
- Reading
- Writing
- Speaking and Listening
- Differentiation

Mathematical Practices
- Overarching Habits
- Reasoning and Explaining
- Modeling and Using Tools
- Seeing Structure and Generalizing
- Differentiation

Creative Problem Solving in Support of MD STEM Standards of Practice
- Understanding Challenges
- Generation of Ideas
- Preparation for Action
- Application of Technology
- Differentiation

Content
- Assessments
- Performance-Based Tasks
- Differentiation

Student Learning Objectives for teachers of grades 4-8 assessed areas – 30%; for others two SLOs from different areas 25%, 25%

Student Learning Objectives for high school teachers of state-assessed courses – one 25% SLO; one two-part SLO that is 10% student performance on state assessment and 15% other teacher-selected data points in alignment with course curriculum

Maryland School Assessments (for teachers grades 4-8 who are teachers of record for mathematics and/or English Language Arts)
## Standards of Student Practice in Mathematics Proficiency Matrix

<table>
<thead>
<tr>
<th>Students:</th>
<th>(I) = Initial</th>
<th>(IN) = Intermediate</th>
<th>(A) = Advanced</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1a</strong> Make sense of problems</td>
<td>Explain their thought processes in solving a problem one way. <em>(Pair – Share)</em></td>
<td>Explain their thought processes in solving a problem and representing it in several ways. <em>(Question/Wait time)</em></td>
<td>Discuss, explain, and demonstrate solving a problem with multiple representations and in multiple ways. <em>(Grouping/Engaging)</em></td>
</tr>
<tr>
<td><strong>1b</strong> Persevere in solving them</td>
<td>Stay with a challenging problem for more than one attempt. <em>(Question/Wait time)</em></td>
<td>Try several approaches in finding a solution, and only seek hints if stuck. <em>(Grouping/Engaging)</em></td>
<td>Struggle with various attempts over time, and learn from previous solution attempts. <em>(Show Thinking)</em></td>
</tr>
<tr>
<td><strong>2</strong> Reason abstractly and quantitatively</td>
<td>Reason with models or pictorial representations to solve problems. <em>(Grouping/Engaging)</em></td>
<td>Are able to translate situations into symbols for solving problems. <em>(Grouping/Engaging)</em></td>
<td>Convert situations into symbols to appropriately solve problems as well as convert symbols into meaningful situations. <em>(Encourage Reasoning)</em></td>
</tr>
<tr>
<td><strong>3a</strong> Construct viable arguments</td>
<td>Explain their thinking for the solution they found. <em>(Show Thinking)</em></td>
<td>Explain their own thinking and thinking of others with accurate vocabulary. <em>(Question/Wait time)</em></td>
<td>Justify and explain, with accurate language and vocabulary, why their solution is correct. <em>(Grouping/Engaging)</em></td>
</tr>
<tr>
<td><strong>3b</strong> Critique the reasoning of others.</td>
<td>Understand and discuss other ideas and approaches. <em>(Pair – Share)</em></td>
<td>Explain other students’ solutions and identify strengths and weaknesses of the solution. <em>(Question/Wait time)</em></td>
<td>Compare and contrast various solution strategies and explain the reasoning of others. <em>(Grouping/Engaging)</em></td>
</tr>
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<tr>
<td>4 Model with Mathematics</td>
<td>Use models to represent and solve a problem, and translate the solution to mathematical symbols. <em>(Grouping/Engaging)</em></td>
<td>Use models and symbols to represent and solve a problem, and accurately explain the solution representation. <em>(Question/Prompt)</em></td>
<td>Use a variety of models, symbolic representations, and technology tools to demonstrate a solution to a problem. <em>(Show Thinking)</em></td>
</tr>
<tr>
<td>5 Use appropriate tools strategically</td>
<td>Use the appropriate tool to find a solution. <em>(Grouping/Engaging)</em></td>
<td>Select from a variety of tools the ones that can be used to solve a problem, and explain their reasoning for the selection. <em>(Grouping/Engaging)</em></td>
<td>Combine various tools, including technology, explore and solve a problem as well as justify their tool selection and problem solution. <em>(Show Thinking)</em></td>
</tr>
<tr>
<td>6 Attend to precision</td>
<td>Communicate their reasoning and solution to others. <em>(Show Thinking)</em></td>
<td>Incorporate appropriate vocabulary and symbols when communicating with others. <em>(Allowing Struggle)</em></td>
<td>Use appropriate symbols, vocabulary, and labeling to effectively communicate and exchange ideas. <em>(Encourage Reasoning)</em></td>
</tr>
<tr>
<td>7 Look for and make use of structure</td>
<td>Look for structure within mathematics to help them solve problems efficiently (such as 2 x 7 x 5 has the same value as 2 x 5 x 7, so instead of multiplying 14 x 5, which is (2 x 7) x 5, the student can mentally calculate 10 x 7. <em>(Question/Prompt)</em></td>
<td>Compose and decompose number situations and relationships through observed patterns in order to simplify solutions. <em>(Allowing Struggle)</em></td>
<td>See complex and complicated mathematical expressions as component parts. <em>(Encourage Reasoning)</em></td>
</tr>
<tr>
<td>8 Look for and express regularity in repeated reasoning</td>
<td>Look for obvious patterns, and use if/then reasoning strategies for obvious patterns. <em>(Grouping/Engaging)</em></td>
<td>Find and explain subtle patterns. <em>(Allowing Struggle)</em></td>
<td>Discover deep, underlying relationships, i.e. uncover a model or equation that unifies the various aspects of a problem such as discovering an underlying function. <em>(Encourage Reasoning)</em></td>
</tr>
</tbody>
</table>

Mathematics Teaching Practices

- Establish mathematics goals to focus learning
- Implement tasks that promote reasoning and problem solving
- Use and connect mathematical representations
- Facilitate meaningful mathematical discourse
- Pose purposeful questions
- Build procedural fluency from conceptual understanding
- Support productive struggle in learning mathematics
- Elicit and use evidence of student thinking

These should be part of every mathematics classroom...

Other Resources

Questions?

For slides and other handouts, visit:
http://www.jonathanwray.com
A New Breakthrough Digital Textbook
DiscoveryEducation.com/ProblemsWorthSolving
Launch Event — January 8, 1PM EST

• Engages students with diverse learning styles by presenting real-world problems that matter to them

• Encourages students to investigate, inquire, and persevere through its instructional approach

• Builds conceptual understanding and procedural fluency
• Allows students to develop understanding with a variety of interactive tools and resources
  
  – Focuses on investigation and problem solving
  
  – Reinforces math concepts and builds procedural fluency
• Supports the educator in delivering effective math instruction and integrating digital curriculum
  
  – Fully embodies the math practices
  
  – Provides teachers with real-time information
  
  – Coupled with on-site professional development
LAUNCH EVENT

January 8, 2015, 1PM EST

Math: What Does it Mean to Win?
Join the live webcast for a panel discussion on math achievement

DiscoveryEducation.com/ProblemsWorthSolving
An on-demand archive of this webinar will be available at www.edweek.org/go/webinar in less than 24 hrs.
Teaching and Measuring the Common Core’s Mathematical Practices

Required Reading from *Education Week*:

**Free download!**

**Spotlight on Math Instruction**

Math instruction is undergoing shifts in the common-core era. In this Spotlight, see how computer-based testing impacts math performance, find out which textbooks are aligned to the common core, and learn how to take a blended learning approach to math instruction.