Algebra Readiness: Equipping K-8 Students for Success
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How to Prepare for Algebra: Our Outline

- **Algebra**: What’s the big deal?
- **Necessary Student Skills**
  - Communication Skills
  - Independent Thinking
- **Essential Fluencies**
  - Fractions & Decimals
  - Variables
  - Algebraic Symbols
\[
\frac{x}{20} = \frac{3}{5}
\]

\[
5x = (20)(3)
\]
\[
\frac{x}{20} = \frac{3}{5}
\]

\[x = 12\]
\[ \frac{3}{20} \times \frac{3}{5} = \frac{3}{10} \div \frac{3}{5} \]
\[
\frac{x}{20} = \frac{3}{5}
\]

\[
\frac{x}{x + 8} = \frac{3}{x - 7}
\]
What is algebra?

The part of mathematics in which letters and other general symbols are used to represent numbers and quantities in formulae and equations.

-Oxford Dictionary

... branches of mathematics or logic concerned with the properties and relationships of abstract entities... manipulated in symbolic form under operations often analogous to those of arithmetic.

–Merriam Webster
The Importance Algebra.

“The best single predictor of successful completion of a STEM degree is whether or not the student studied calculus in high school.”

-Chen, X. Students who study Science, Technology, Engineering, and Mathematics (STEM) in Postsecondary Education, 2009

…and to be ready for calculus, students must start investigating algebra before high school.
The main purpose of algebra is to learn how to represent **general relationships and procedures**; for through these representations, a wide range of problems can be solved and new relationships can be **developed from those known**…

However, students tend to view algebra as little more than a set of **arbitrary** manipulative techniques that seem to have **little, if any, purpose** to them.

— L.R. Booth. Difficulties in Algebra. 1986
In order for students to succeed in Algebra… students must learn to think algebraically.

This focus can start and continue through all grades, from kindergarten to eighth.
Because Algebra has come to be regarded as a gatekeeper ... the high failure rate in Algebra, especially among minority students, has rightfully become an issue of general social concern.

– H. Wu, math.berkeley.edu
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So, focus on algebra... easy, right?

Algebra I is the **key** — and the **barrier** — to students’ ability to complete a challenging mathematics curriculum in high school.

- Southern Regional Education Board, publications.sreb.org
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Over 60 percent of all students entering community colleges must take what are called developmental math courses … [that] are algebra-based and focus on linear and quadratic equations.

- Ginia Bellafante, nytimes.com, 2014
What do students need to be ready to do in Algebra?

Perform calculations and solve basic equations?
Low-level skills

If a student feels that she or he has learned nothing that cannot be pulled directly from Wolfram|Alpha, then the course really has been a waste of time.

- David M. Bressoud, maa.org, 2009
Because [low-achieving students] are less likely to have acquired the basics on the same schedule as more advanced learners, struggling learners are often confined to an educational regimen of low-level activities, rote memorization of discrete facts, and mind-numbing skill-drill worksheets… [They] have minimal opportunities to actually use what they are learning in a meaningful fashion.

- Wiggins & McTighe, Schooling by Design

**Discrete Low-level skills**
Prerequisite 1

Communication Skills
Communication Skills

Algebra is as much about constructing arguments as it is about simplifying and solving.

A crate can hold 24 cubic boxes, each with sides one meter in length. List the different possible dimensions of the crate. How do you know when you’ve found every case?

… Maria uses the equation $P(x) = 250x - 900$ to represent the profit she has made. Identify the rate of change and explain its meaning in this context.
Identify the x-intercepts for the function \( f(x) = x^2 - 3x - 10 \). Record your method for determining the values, and state why this method is more efficient than other possible methods.

... all employee salaries are recorded in the table above. Determine the mean, the median and the mode. If a representative meant to request a employee-wide raise, which measure of central tendency would he use to support his argument?
A local bus rental company charges a daily fee of $500, plus $50 per bus used. What will the total cost be to rent 14 buses for one day?

\[ T(x) = 50x + 500 \]

\[ T(14) = 50(14) + 500 \]

\[ T(14) = 1200 \]

Multiply $50 by 14 buses.

\[ 50(14) = 700 \]

Add 700 to the daily fee of 500.

\[ 700 + 500 = 1200. \]

<table>
<thead>
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<th>Buses</th>
<th>0</th>
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<th>2</th>
<th>4</th>
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<td>550</td>
<td>600</td>
<td>700</td>
<td>…</td>
<td>1200</td>
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What are the strengths and weaknesses of each method? What do they tell you about what a student would do outside of the class right now?
Prerequisite 2

Independent Thinking
Independent Thinking

Do your students think about math like they do art? Or language? Can they express with it? Can they create with it? Can they stretch the rules?
Independent Thinking

Don't ever let kids say, “I do this because it's what my teacher did.”

Is it more important to copy a teacher? Or to own their own strategy and be comfortable with their own explanation?
I’d like to determine the value of $x$, what might I do first?

“Rewrite five halves as a fraction with eighths.”

“Multiply both sides of the equation by eight.”

“Can we just guess values for $x$ and check?”

“Split the right side into two separate fractions.”

Okay, let’s discuss each of those options.
Again, Cross Multiplication

\[ \frac{x}{20} = \frac{3}{5} \]

**Nix the Tricks**

“I would say, then, that it is not reasonable to even mention this technique. If it is so limited in its usefulness, why grant it the privilege of a name and some memory space…”

-Jim Doherty
Prerequisite Essential Fluencies for Algebra

Fluency with **integers, fractions, decimals, percents**.

Fluency with **algebraic equations, expressions, and symbols**.

Factors and Multiples.

Ratios, rates, proportions.

Geometric properties.

Represent/interpret data.

Represent/analyze patterns.

-Rachael M. Welder, Prerequisite Knowledge for Learning of Algebra, 2006
Prerequisite 3

Fluency with Rational Numbers
Prerequisite 3: Fractions and Decimals

Once in Algebra will a student be familiar enough with fractions to answer these questions?

17 = 3x
How many ways can you represent the solution to this equation?

What are the benefits of the forms: improper fraction, mixed number or decimal?

\[ \frac{1}{3} n + 2 = \frac{5}{6} n + 12 \]
What is an efficient way to begin solving this equation?

\[ \frac{x-2}{x+2} + \frac{x+3}{x-2} = 5 \]
How might I go about solving for x in this case?
Prerequisite 4

Fluency with Algebraic Symbols… including variables
Good Ol’ Cross Multiplication

\[
\frac{a}{b} = \frac{c}{d} \quad \Rightarrow \quad \frac{ad}{bd} = \frac{bc}{bd}
\]

\[
ad = bc
\]
“Children should first learn how to find and record patterns and write pattern-rules in words. Eventually they will seek more concise ways of writing rules. At this time, the introduction of variables will make sense and be appreciated by the student.”

-Rachael M. Welder, Prerequisite Knowledge for Learning of Algebra, 2006
Prerequisite 4: Fluency with Variables

Solve $5 \times 60$ by entering the product in the equation.

$25 \times 68 = ?$

$(5 \times 60) =$ [ ] [ ]

$5 \times 60 = \boxed{300}$
Prerequisite 4: Fluency with Variables

Calculate this product to help you solve the problem.

\[(7 + b) \times 45\]

\[7 \times 40 = \square\]
Prerequisite 4: Fluency with Variables

Create an array that represents this expression.

$$(x + 7y)(x + 7y)$$
More Prerequisite 4

Fluency with *other* Algebraic Symbols
Prerequisite 5: Fluency with Symbols

3 + 4
“Simplify this term to one value.”

2x + 12
“Can this be simplified?”

12 + 8 = [ ]
“Simplify this expression and write the answer on the right.”
12 + 8 = 20 + 10

Do students understand “expressions on either side of the equation are equivalent?”

Concatenation
2x vs 2\frac{1}{2}
What does concatenation mean in each case?
Algebra or algebra in the 8th grade?

Many school districts are aiming to have students complete Algebra 1 before high school.

As we accelerate towards this goal, remember that the foundation can be laid early. The preceding years can rigorously build confidence with rational numbers, symbolic reasoning and independent thinking.
Summary: Prerequisites to Algebra

- Communication Skills
- Independent Thinking
- Fluency with integers, fractions and decimals.
- Fluency with algebraic symbols...
  - Including variables
  - Including other symbols common to expressions and equations
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Intelligently adapt & individualize to:

• Students’ own intuitive strategies
• Kinds of mistakes
• Efficiency of strategy
• Scaffolding needed
• Response time
## Actionable Reporting

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<thead>
<tr>
<th>Student</th>
<th>Grade</th>
<th>Kindergarten Curriculum</th>
<th>1st Grade Curriculum</th>
<th>2nd Grade Curriculum</th>
<th>3rd Grade Curriculum</th>
<th>Time on Task (HH:MM)</th>
<th>Notifications</th>
<th>Student Reports</th>
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**Concept:** Multiplication: Double & Half

Students use known basic facts and double one factor and halve the other to determine the product of a more challenging problem.

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<th># Completed with Proficiency</th>
<th># In Progress</th>
<th># Not Started</th>
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<td>Avaneesh S (71%)</td>
<td>Anthony P</td>
</tr>
<tr>
<td>Jacob C</td>
<td>(about 1 month ago)</td>
<td>Charles K (71%)</td>
<td>Brittany B</td>
</tr>
<tr>
<td>Rebecah D</td>
<td>(about 1 month ago)</td>
<td>Emmanuel M (71%)</td>
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<td></td>
<td>Suna C (28%)</td>
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<td>Caitlin S (21%)</td>
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