Digging Into Mathematical Discourse

Selecting and Sequencing Student Solution Samples

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Digging into Mathematics Discourse:
Selecting and Sequencing Student Solution Samples

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Goals

“To consider ways to engage students in meaningful and productive mathematics discussions”

• Why is task selection important?
• How do I select, sequence, and discuss tasks to support mathematics learning?
• How do I encourage students to represent solutions in mathematically appropriate ways?
• How do I encourage students to examine and critique work samples to enhance learning?

Like a music conductor, the teacher plays a critical role in arranging, coordinating, and managing the flow of the mathematics discourse.
“Mathematical discourse serves as a forum in which to exchange and record mathematics ideas, document evidence of learning, and monitor growth in knowledge development.

Teachers gain insights not only about what students know but also about the approaches they use, how and how well they understand the ideas, and the ways they present their knowledge.”
Task Selection

Task 1:
Find the mean, median, and mode of the following set of numbers: 6, 5, 5, 5, 7, 9, 11

Task 2:
Create a data set with at least 7 values so that the mean is 20, the median is 12, and the mode is 23.
### Four Levels of Cognitive Demand in Mathematical Tasks

#### Low-Level Cognitive Demands

- Memorization Tasks
- Procedures *Without* connections to understanding, meaning, or concepts/tasks

#### High-Level Cognitive Demands

- Procedures *With* connections to understanding, meaning, or concepts/tasks
- Doing mathematics tasks

Weblink: [Mathematical Task Analysis Guide](https://example.com) (Stein and Smith, 1998)
Planning and Leading Mathematics Discussions

- Anticipate
- Monitor
- Select
- Sequence
- Connect
## Representing and Communicating Mathematics Solutions

### Representation

- Physical materials
- Pictures
- Diagrams or charts
- Words
  - Oral and written
  - Everyday and mathematics-specific
- Symbols

### Communication

- Relate the representations to mathematically appropriate ideas
- Use appropriate mathematics diagrams, language, and symbols to convey intended ideas
Recognizing Student Attempts at Conveying Mathematical Knowledge

1. Ordinary Language
   - “2 boys for each girl”
2. Mathematics Verbal Language
   - “Twice as many boys as girls”
   - \[2G = B\]
3. Symbolic Language
4. Visual Representation
5. Unspoken Shared Assumptions
6. Quasi-mathematical Language
Questions ???

Language of Mathematics

Explain and Critique

Deeper Understanding
An Example

Task

“Folding chairs are set up in a school auditorium for a play. There are 16 rows of chairs, each with 28 chairs. How many folding chairs are there?”

Questions to consider

- What do you hope to learn about students’ understanding?
- What is your purpose for using this problem?
  - Develop a new concept
  - Reinforce what’s already learned
- What behaviors, norms, or routines are you hoping to reinforce?
- What are the communication goals for the lesson (use of math language, expression of mathematics ideas, etc.)
Student Solution Samples

28 x 16 = 448

You can use multiples:
1 x 28 = 28
2 x 28 = 56
3 x 28 = 84
4 x 28 = 112
5 x 28 = 140
6 x 28 = 168
7 x 28 = 205
8 x 28 = 232
9 x 28 = 259
10 x 28 = 288
11 x 28 = 316
12 x 28 = 344
13 x 28 = 372
14 x 28 = 400
15 x 28 = 428
16 x 28 = 456

28 x 10 = 280
28 x 6 = 168
280 + 168 = 448

16 x 28 =

\[ \frac{280}{280} + \frac{168}{168} = 448 \]
Rationale: Examine Use of Visual Representations

Taylor

Joaquin

Francis

Raul

Robin
Rationale: Examine How Students are Using Place Value Understandings

Juan

\[
\begin{array}{c|c|c|c}
10 & 200 & 80 \\
\hline
6 & 120 & 48 \\
\hline
20 + 8 & & \\
\hline
200 & 200 & 80 \\
\hline
120 & 120 & 48 \\
\hline
48 & 48 & \\
\hline
\end{array}
\]

Rashid

\[
\begin{array}{c}
16 \\
\times 28 \\
\hline
48 \\
\hline
80 \\
\hline
120 \\
\hline
448 \\
\end{array}
\]

Angel

\[
\begin{align*}
16 \times 28 &= 448 \\
28 \times 10 &= 280 \\
28 \times 6 &= 168 \\
280 + 168 &= 448
\end{align*}
\]
Rationale: Expose Students to “chunking” as a Strategy

Aisha

\[
\begin{align*}
3 \\
28 \\
+28 \\
+28 \\
+28 \\
112 \\
\times 4 \\
448
\end{align*}
\]

Jordan

\[
\begin{align*}
16 & \quad 16 & \quad 16 & \quad 16 \\
\times 7 & \quad 7 & \quad 7 & \quad 7 \\
\hline
112 & +112 & +112 & +112 \\
\hline
148 & & & \\
\div 28 &= 5
\end{align*}
\]
Rationale: Compare/Contrast Similar Approaches

Lei

28 x 16 = 448
You can use multiples
1 28
2 56
3 84
4 112
5 140
6 168
7 196
8 224
9 252
10 280
11 308
12 336
13 364
14 392
15 420
16 448

Raul

Row 1 | 28 chairs
Row 2 | 28 chairs
Row 3 | 28 chairs
Row 4 | 28 chairs
Row 5 | 28 chairs
Row 6 | 28 chairs
Row 7 | 28 chairs
Row 8 | 28 chairs
Row 9 | 28 chairs
Row 10 | 28 chairs
Row 11 | 28 chairs
Row 12 | 28 chairs
Row 13 | 28 chairs
Row 14 | 28 chairs
Row 15 | 28 chairs
Row 16 | 28 chairs
Questions ???
Questions to Consider ... 

When selecting student solution samples

• What patterns am I noticing in students’ solutions?
• What do the identified patterns represent in terms of students’ mathematical understanding?
• What are the unique strategies or approaches used by students? Will they always work?
• What mathematics ideas can I highlight or reinforce as a result of bringing this example to the class’s attention?
• What errors do I notice? How can I use these errors as springboards for additional learning?
Questions to Consider ...

When selecting which students to call on

• How do I ensure that every student has a voice in my classroom?

• What supports do I need to provide to prepare students to experience success in sharing their ideas?

• How do I structure the learning environment so that students are willing to expose their mistakes and, as a result, learn from them?

• How do I make math conversations a regular part of classroom interactions?
Questions to Consider ...

When sequencing student solution samples

• What are the advantages/disadvantages of sequencing the discussion in this order?

• How do I build students’ understanding as we transition from one set of solution strategies to the next?

• What questions do I ask to encourage students to critique ideas that are presented?

• How do the questions support students in making connections between and among the work to be shared?
Making Connections: Pulling It All Together

28 x 16 = 448

28 x 28 chairs
araaraaraara
araaraaraarata = 448
araaraarata =

16 x 28 =

\[ \frac{28 \times 10}{28 \times 6} = \frac{280}{168} = 448 \]

28 chairs
Row 1
Row 2
Row 3
Row 4
Row 5
Row 6
Row 7
Row 8
Row 9
Row 10
Row 11
Row 12
Row 13
Row 14
Row 15
Row 16

0 = 4 chairs

32

4 chairs

4 x 16 = 448
Ask Questions to Promote Discourse

1. Encourage students to listen to each other.
   - Mark, can you repeat what Sally said?
   - Tell your partner what you think the answer is and why.

2. Compare and contrast different solution strategies.
   - What do these have in common?
   - How are they different?
   - When will this work?
   - Will this work in all cases?

3. Ask students to clarify or restate peers' comments.
   - Can someone say that differently?

4. Use the tools of mathematical discourse to help students present their ideas.
   - Is there a way to show that visually?
   - What mathematics words can you use to express that idea?

5. Teach students to respectfully critique the reasoning of their peers and disagree amicably. Give them some examples.
   - I disagree with Bena because...
   - You said X, but I think you meant to say Y.

6. Use the language of mathematics.
   - Would you say that again using some of the mathematics words we've been learning?
Questions ???

Language of Mathematics

Explain and Critique

Deeper Understanding
Contact Information

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