Selecting and Sequencing Student Solutions For Productive Math Discourse
Goals

To consider ways to engage students in meaningful and productive mathematics discussions

• Why is task selection important?
• How do I select, sequence, and discuss tasks to support mathematics learning?
• How do I encourage students to represent solutions in mathematically appropriate ways?
• How do I encourage students to examine and critique work samples to enhance learning?

Like a music conductor, the teacher plays a critical role in arranging, coordinating, and managing the flow of the mathematics discourse.
“Mathematical discourse serves as a forum in which to exchange and record mathematics ideas, document evidence of learning, and monitor growth in knowledge development.

Teachers gain insights not only about what students know but also about the approaches they use, how well they understand the ideas and the ways they present their knowledge.”
Task Selection

Task 1:
Find the **mean**, **median**, and **mode** of the following set of numbers: 6, 5, 5, 5, 7, 9, 11

Task 2:
Create a data set with at least 7 values so that the **mean** is 20, the **median** is 12, and the **mode** is 23.
### Four Levels of Cognitive Demand in Mathematical Tasks

<table>
<thead>
<tr>
<th>Low-Level Cognitive Demands</th>
<th>High-Level Cognitive Demands</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Memorization Tasks</td>
<td>• Procedures <em>With</em> connections to understanding, meaning, or concepts/tasks</td>
</tr>
<tr>
<td>• Procedures <em>Without</em> connections to understanding, meaning, or concepts/tasks</td>
<td>• Doing mathematics tasks</td>
</tr>
</tbody>
</table>

Weblink: [Mathematical Task Analysis Guide](http://example.com) (Stein and Smith, 1998)
Planning and Leading Mathematics Discussions

- Anticipate
- Monitor
- Select
- Sequence
- Connect
Representing and Communicating Mathematics Solutions

**Representation**

- Physical materials
- Pictures
- Diagrams or charts
- Words
  - Oral and written
  - Everyday and mathematics-specific
- Symbols

**Communication**

- Relate the representations to mathematically appropriate ideas
- Use appropriate mathematics diagrams, language, and symbols to convey intended ideas

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Recognizing Student Attempts at Conveying Mathematical Knowledge

1. Ordinary Language
   - “2 boys for each girl”
   - “Twice as many boys as girls”
   \[2G = B\]

2. Mathematics Verbal Language
3. Symbolic Language
4. Visual Representation
5. Unspoken Shared Assumptions
6. Quasi-mathematical Language
Questions ???

LANGUAGE OF
MATHEMATICS

EXPLAIN AND
CRITIQUE

DEEPER
UNDERSTANDING

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An Example

Task

Folding chairs are set up in a school auditorium for a play. There are 16 rows of chairs, each with 28 chairs. How many folding chairs are there?

Questions to consider

• What do you hope to learn about students’ understanding?

• What is your purpose for using this problem?
  – Develop a new concept
  – Reinforce what’s already been learned

• What behaviors, norms, or routines are you hoping to reinforce?

• What are the communication goals for the lesson (use of math language, expression of mathematics ideas, etc.)
Student Solution Samples

2018
200 80
80
48

16
38

16
48
120
120
448

2 a = 28 chairs
at a time
at a time
= 448

28 x 16 = 448

28
16
28
16
16
16
16
16
28

112

7
7
7
7

16 x 28 =

280
280

280 + 168 = 448

16 x 28 =
Rationale: Examine Use of Visual Representations

Several students visually illustrated the problem using discrete or area model representations.

<table>
<thead>
<tr>
<th>Robin</th>
<th>Francis</th>
<th>Joaquin</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Robin's drawing" /></td>
<td><img src="image2" alt="Francis' drawing" /></td>
<td><img src="image3" alt="Joaquin's drawing" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Taylor</th>
<th>Raul</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image4" alt="Taylor's drawing" /></td>
<td><img src="image5" alt="Raul's drawing" /></td>
</tr>
</tbody>
</table>
Rationale: Examine Students Using Place Value Understanding

Some students used their understanding of place value to find a solution.

Juan

Angel

Raashid

\[
\begin{align*}
16 \times 28 &= 448 \\
28 \times 10 &= 280 \quad \frac{4}{28} \\
28 \times 6 &= 168 \quad \frac{4}{28} \\
280 + 168 &= 448
\end{align*}
\]
Rationale: Expose Students to “chunking” as a Strategy

Jordan

\[
\begin{align*}
16 & \times 7 \\
16 & \times 7 \\
16 & \times 7 \\
16 & \times 7 \\
\hline
112 + 112 + 112 + 112 &= 448 \div 28 \\
\end{align*}
\]

Aisha

\[
\begin{align*}
3 \\
28 + 28 \\
+ 28 \\
+ 28 \\
\hline
112 \\
x 4 \\
\hline
448
\end{align*}
\]
Rationale: Compare/Contrast Similar Approaches

Lei

\[ 28 \times 16 = 448 \]

You can use multiples

<table>
<thead>
<tr>
<th>1</th>
<th>28</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>56</td>
</tr>
<tr>
<td>3</td>
<td>84</td>
</tr>
<tr>
<td>4</td>
<td>112</td>
</tr>
<tr>
<td>5</td>
<td>140</td>
</tr>
<tr>
<td>6</td>
<td>168</td>
</tr>
<tr>
<td>7</td>
<td>196</td>
</tr>
<tr>
<td>8</td>
<td>224</td>
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<tr>
<td>9</td>
<td>252</td>
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<tr>
<td>10</td>
<td>280</td>
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<tr>
<td>11</td>
<td>308</td>
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<tr>
<td>12</td>
<td>336</td>
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<td>14</td>
<td>392</td>
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<tr>
<td>15</td>
<td>420</td>
</tr>
<tr>
<td>16</td>
<td>448</td>
</tr>
</tbody>
</table>

Raul

<table>
<thead>
<tr>
<th>Row 1</th>
<th>28 chairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row 2</td>
<td>28 chairs</td>
</tr>
<tr>
<td>Row 3</td>
<td>28 chairs</td>
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<tr>
<td>Row 5</td>
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<td>Row 6</td>
<td>28 chairs</td>
</tr>
<tr>
<td>Row 7</td>
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<tr>
<td>Row 8</td>
<td>28 chairs</td>
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<tr>
<td>Row 9</td>
<td>28 chairs</td>
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<tr>
<td>Row 10</td>
<td>28 chairs</td>
</tr>
<tr>
<td>Row 11</td>
<td>28 chairs</td>
</tr>
<tr>
<td>Row 12</td>
<td>28 chairs</td>
</tr>
<tr>
<td>Row 13</td>
<td>28 chairs</td>
</tr>
<tr>
<td>Row 14</td>
<td>28 chairs</td>
</tr>
<tr>
<td>Row 15</td>
<td>28 chairs</td>
</tr>
<tr>
<td>Row 16</td>
<td>28 chairs</td>
</tr>
</tbody>
</table>
Questions ???

\[
\begin{align*}
\text{16 + 120} & = 136 \\
28 & = 28 \\
+28 & = 56 \\
+28 & = 84 \\
+28 & = 112 \\
\frac{112}{4} & = 28 \\
\frac{28}{4} & = 7 \\
\frac{7}{4} & = 1.75 \\
+20 & = 48 \\
\frac{120}{8} & = 15 \\
\frac{120}{4} & = 30 \\
\frac{120}{48} & = 2.5 \\
\end{align*}
\]
Questions to Consider ...

When selecting student solutions

• What patterns am I noticing in students’ solutions?
• What do the identified patterns represent in terms of students’ mathematical understanding?
• What are the unique strategies or approaches used by students? Will they always work?
• What mathematics ideas can I highlight or reinforce as a result of bringing this example to the class’s attention?
• What errors do I notice? How can I use these errors as springboards for additional learning?
Questions to Consider ...

When selecting which students to call on

• How do I ensure that every student has a voice in my classroom?

• What supports do I need to provide to prepare students to experience success in sharing their ideas?

• How do I structure the learning environment so that students are willing to expose their mistakes and, as a result, learn from them?

• How do I make math conversations a regular part of classroom interactions?
Questions to Consider ...

When sequencing student solutions

• What are the advantages/disadvantages of sequencing the discussion in this order?
• How do I build students’ understanding as we transition from one set of solution strategies to the next?
• What questions do I ask to encourage students to critique ideas that are presented?
• How do the questions support students in making connections between and among the work to be shared?
Making Connections: Pulling It All Together

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Ask Questions to Promote Discourse

Encourage students to listen to each other.

- Mark, can you repeat what Sally said?
- Tell your partner what you think the answer is and why.

Ask students to clarify or restate peers’ comments.

- Can someone say that differently?

Teach students to respectfully critique the reasoning of their peers and disagree amicably. Give them some examples.

- I disagree with Bena because...
- You said X, but I think you meant to say Y.

Compare and contrast different solution strategies.

- What do these have in common?
- How are they different?
- When will this work? Will this work in all cases?

Use the tools of mathematical discourse to help students present their ideas.

- Is there a way to show that visually?
- What mathematics words can you use to express that idea?

Use the language of mathematics.

- Would you say that again using some of the mathematics words we’ve been learning?
Questions ???

\[
\begin{align*}
28 \\
+ 28 \\
+ 28 \\
+ 28 \\
\hline
112 \\
\times 4 \\
\hline
448
\end{align*}
\]

\[
\begin{array}{ccc}
10 & 20 & 8 \\
+ 6 & 80 & 48 \\
\hline
200 & 80 & 4 \quad 8
\end{array}
\]

\[
\begin{align*}
16 \\
\times 28 \\
\hline
48 \\
80 \\
120 \\
200 \\
\hline
448
\end{align*}
\]
Contact Information

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