What Happens When Teachers Talk Less And Students Talk More In the Mathematics Classroom?
What Happens When Teachers Talk Less and Students Talk More?

Using instructional routines that develop productive habits for success.
Join the Conversation!

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#MathRoutines
Mark Ellis, Ph.D., NBCT

Professor of Education at California State University, Fullerton
Examining Student Work

The sum of $\frac{1}{12}$ and $\frac{7}{8}$ is closest to

A. 20
B. 8
C. $\frac{1}{2}$
D. 1

What does this tell us about the student’s understanding?

Explain your answer.

\[
\frac{1}{12} + \frac{7}{8} = \frac{2}{24} + \frac{21}{24} = \frac{23}{24}
\]
Examining Student Work

The sum of \( \frac{1}{12} \) and \( \frac{7}{8} \) is closest to

A. 20
B. 8
C. \( \frac{1}{2} \)
D. 1

Explain your answer.

\[
\frac{1}{12} + \frac{7}{8} = \frac{2}{24} + \frac{21}{24} = \frac{23}{24} \text{ is closest to } 20.
\]

What does this tell us about the student’s understanding?
Public Perception of Math

‘We hate math,’ say 4 in 10 — a majority of Americans

WASHINGTON — People in this country have a love-hate relationship with math, a favorite school subject for some but just a bad memory for many others, especially women. In an AP-AOL News poll as students head back to school, almost four in 10 adults surveyed said they hated math in school, a widespread disdain that complicates efforts today


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Guiding Questions

• What sort of **routines of doing math** have students developed as a result of 20th century teaching practices?
• What are more productive routines that support reasoning and sense making?
• What are some of the instructional **practices** that support 21st century routines?
Focus of US Math Lessons

Avg % of problems per lesson


Giving Results only
Using Procedures
Stating Concepts
Making Connections

Australia  Czech Republic  Hong Kong SAR  Japan  Netherlands  United States
“What is 35 + 27?”

[excited, waving hand]

62

“Very Good.”

The I-R-E questioning pattern contributes to the undesirable outcomes we discussed earlier.
20th Century Instructional Routine

I do.  We do.  You do.
20th Century Instructional Routine

I do.  We do.  You do.

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Mathematical Groundhog Day

• “What if there is no tomorrow?”

• We must consider our students’ mathematical tomorrows.

• This requires careful attention to the routines that govern students’ opportunities to develop productive habits and deep understandings.
What Should 21st Century Mathematics Instruction Look Like?
Research Tells Us...

- Mathematics aptitude is primarily a function of opportunity, experience, and effort—not innate intelligence.
- Effective mathematics teaching cultivates mathematics abilities.
- Equitable access and support includes attention to students’ reasoning and identities—*one size does not fit all.*

New Expectations for Learning Mathematics

• Understand the concepts behind the calculations
• Skill in reasoning and communicating mathematically
• Flexibility to solve non-routine problems

How does this look in the classroom?
Habits in Mathematics Classrooms Today: Standards for Mathematical Practice

Mathematically proficient students...

1. Make sense of problems and **persevere** in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate **tools** strategically.
6. Attend to **precision**.
7. Look for and make use of **structure**.
8. Look for and express regularity in **repeated reasoning**.

#MathRoutines
Deep Learning Requires 
*Active Cognition*

“Students need enough freedom to become cognitively active in the process of sense making...”

“and students need enough guidance so their cognitive activity results in the construction of useful knowledge.”

(Mayer, 2004, p. 16)
Problem Solving Followed by Instruction

a) “Students activate their prior knowledge during the problem-solving phase;”

(Loibl, 2013, p. 133)
Problem Solving Followed by Instruction

a) “Students activate their prior knowledge during the problem-solving phase;

b) as they struggle with the problem during this phase, they become aware of their knowledge gaps, which prepares them to…

(Loibl, 2013, p. 133)
Problem Solving Followed by Instruction

a) “Students activate their prior knowledge during the problem-solving phase;

b) as they struggle with the problem during this phase, they become aware of their knowledge gaps, which prepares them to…

c) understand the rationale behind the solution taught in the subsequent instruction phase by decomposing the solution into its functional components.”

(Loibl, 2013, p. 133)
Routine That Supports Deep Learning

Think
1. Make sense of the problem
2. Solve and support your thinking

Share
3. Share strategies with a partner (pair/share)
4. Share strategies as a class (whole class)

Compare
5. Make connections between strategies (students and text)
6. Apply strategies to new problems

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Routine That Supports Deep Learning

Think

1. Teacher facilitates a conversation where students make sense of the problem and identify important information.

2. Use existing knowledge and intuitive ideas to think about strategies for solving a problem that involves a new relationship or concept.
Routine That Supports Deep Learning

**Share**

3. **Share** thinking among peers to get feedback and additional insights; become aware of gaps, incomplete understandings.

4. **Share** thinking with the class, with multiple students explaining their strategies and other students politely critiquing the reasoning of others.
Routine That Supports Deep Learning

**Compare**

5. Students **compare** and make connections between strategies they developed and to standards-aligned strategies to clarify and extend understanding.

6. Students **apply** and connect what they have learned to new problems.
Let’s See the Routine in Action
Think Make Sense of the Problem

- What is the problem about?
- What are you trying to find out?
- What information is important?

Problem Think about ways to solve the problem.

How many marbles?

35 marbles

27 marbles
Step 2  **THINK Solve and Support Your Thinking**

- Solve the problem.
- Record your process.
- Solve another way.

**Problem** Think about ways to solve the problem.

**How many marbles?**

- 35 marbles
- 27 marbles

Grade 5  Introduction
SHARE: Discuss It

• Share your thinking with a partner.

Discussion Starters
Use these to start sharing your thinking.

• The strategy I used to find the answer was . . .

• Do you agree with me? Why or why not?

• Why did you choose that strategy?
### Step 4

**SHARE: Discuss It**

- Discuss class strategies

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**How many marbles?**

**35 marbles**

**27 marbles**

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- **35**
  - **30**
  - **5**

- **27**
  - **20**
  - **7**

---

\[
\begin{align*}
30 + 20 &= 50 \\
5 + 7 &= 12 \\
50 + 10 + 2 &= 62 \\
35 + 55 + 60 &= 150 \\
&= 62
\end{align*}
\]
Adding Two-Digit Numbers with Regrouping

\[
\begin{align*}
5 + 7 &= 12 \\
30 + 20 &= 50 \\
50 + 12 &= 62 \\
35 + 27 &= 62
\end{align*}
\]
**COMPARE:** Connect It

- Connect student strategies.
- Connect to standards-aligned strategies.

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**Model It**  
Find $35 + 27$.

Add the tens and ones.

$\begin{align*}
\text{5 tens} &\quad \text{12 ones} \\
50 &\quad + \\
50 &\quad + \\
\end{align*}$

$35 + 27 = \underline{____}$
There are 36 white eggs and 25 brown eggs. How many eggs?
Questions?
Takeaways

• Tasks matter
  – Clear connection to prior knowledge
  – Elasticity in the strategies students might use
  – Potential to surface gaps in understanding or knowledge

**Aspects of Rigor in Assignments**

<table>
<thead>
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<th>Conceptual Understanding</th>
<th>Procedural Skills and Fluency</th>
<th>Application</th>
<th>Multiple Representations</th>
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<tr>
<td>38%</td>
<td>87%</td>
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**Conceptual Understanding**
Students access concepts from a number of perspectives in order to see math as more than a set of mnemonics or discreet procedures.

**Procedural Skills and Fluency**
Students have speed and accuracy in calculation in order to have access to more complex concepts and procedures.

**Application**
Students use math in situations that require mathematical knowledge.


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Ms. Stevens drew a picture on the board:

She then asked her students to shade \( \frac{2}{6} \) of the picture.
Cooper drew the models below. He says they show $\frac{2}{3} = \frac{2}{6}$.

How could Cooper’s statement make sense?
How could Cooper’s statement be considered incorrect?
Takeaways

• **Tasks matter**
  – Clear connection to prior knowledge
  – Elasticity in the strategies students might use
  – Potential to surface gaps in understanding or knowledge

• **Teacher’s role is essential**
  – Confirm that students understand the task
  – Support productive student engagement
  – Pay attention to status issues
  – Select and sequence student work to share (see Dr. Kersaint’s whitepaper)
  – Call attention to important mathematical features, relationships
Takeaways

• **Tasks matter**
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  – Select and sequence student work to share (see Dr. Kersaint’s whitepaper)
  – Call attention to important mathematical features, relationships

• After student exploration, help students organize, articulate, and apply their learning.
Management Tips

Allow time for productive struggle.

- Have students talk with a partner about their ideas or try another strategy.
- Avoid telling students whether their approach or answers are incorrect. Instead, prompt students:
  - Do you agree with [student’s] answer? Why or why not?

Create classroom discussion guidelines.

- Have students listen carefully to their partners.
- Tell students to ask questions if they don’t understand, such as I was confused when you said…
- Guide students to respect others’ ideas and add on to them.

Prepare for the classroom conversations.

- Circulate as students work.
- Identify strategies and models to highlight in the classroom discussion.
- Sequence the strategies you want to discuss. You may want to start with a student who has an incorrect answer or solution process to address common misconceptions and promote discussion.
What can you do now?

- Open up tasks that are more narrow
- Have students do the thinking and explaining
- Ask students to restate what another student said
- Have students turn and talk to a partner using sentence starters or discourse questions
- Have students determine if they agree or politely disagree with other students’ responses
- Honor student questions and comments
- Have students describe their strategies for someone else to record, rather than just “show” their strategies.
Teaching Practices to Support Standards for Mathematical Practice

NCTM Teaching Practices

1. Establish mathematical goals to focus learning
2. Implement tasks that promote reasoning and problem solving
3. Use and connect mathematical representations
4. Facilitate meaningful mathematical discourse
5. Pose purposeful questions
6. Build procedural fluency from conceptual understanding
7. Support productive struggle in learning mathematics
8. Elicit and use evidence of student thinking

(NCTM, 2014)
Connecting Teaching Practices to Think–Share–Compare routine

- **Eliciting Student Thinking**, including providing opportunities for students to generate ideas and then share their ideas with the class;
- **Supporting Student-to-Student Exchanges about Mathematical Ideas**, including establishing a common knowledge base from which to work and helping students make sense of one another’s ideas;
- **Guiding and Extending the Math**, including guiding the development of mathematical ideas, pursuing common misconceptions and ensuring appropriate disciplinary norms to advance the learning of the class.

Staples & King, 2017
https://www.nctm.org/Store/Products/Enhancing-Classroom-Practice-with-Research-behind-Principles-to-Actions/
Specific Routines That Support Think–Share–Compare

https://www.heinemann.com/products/e07815.aspx
http://www.fosteringmathpractices.com
Closing Thoughts

• Think–Share–Compare is flexible
  – Can be used for short tasks or longer investigations
  – Find tasks to use inside of it, such as Number Talks, Always/Sometimes/Never, and Error Analysis

• Learning requirements
  – Active engagement
  – Problem-solving that makes connections to prior knowledge
  – Students have time to generate, discuss, and compare strategies
  – Students have opportunities to refine understanding and make sense of gaps in their learning with teacher guidance.

• Teachers are learners, too.
Thank you!

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ReadyMathematics.com/EWwroutines

Questions?

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References


